Exercise 319

Find the degree, y-intercept, and zeros for the following polynomial functions.

$$f(x) = x^3 + 2x^2 - 2x$$

Solution

The degree is the highest power of x in the polynomial.

Degree: 3

The y-intercept is the point where the curve crosses the y-axis. To find the y-value, plug in x = 0.

$$f(0) = (0)^3 + 2(0)^2 - 2(0) = 0$$

Therefore, the y-intercept is (0,0). The zeros are values of x where f(x) = 0.

$$f(x) = x^{3} + 2x^{2} - 2x = 0$$

$$x(x^{2} + 2x - 2) = 0$$

$$x = 0 \quad \text{or} \quad x^{2} + 2x - 2 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{4 - 4(1)(-2)}}{2(1)} = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}$$

The zeros are then

$$x = \{-1 - \sqrt{3}, 0, -1 + \sqrt{3}\}.$$